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STABILIZATION OF LINEAR SYSTEMS WITH MULTIPLICATIVE PERTURBATIONS AND INCOMPLETE INFORMATION*

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The problem of stability of a stochastic system with several multiplicative perturbations is reduced to the determination of a quadratic criterion for a simpler system with a lesser number of perturbations. A sequential procedure that makes possible the determination of the necessary and sufficient conditions of stability and, also, of the stabilizing control parameters is proposed for a controlled system with multiplicative perturbations and incomplete information. The problem is reduced to solving a number of optimalization problems. By majorizing several perturbations by a single one a theorem on merging is obtained, which provides sufficient conditions of stability that considerably simplifies construction of the stabilizing control.

The present investigation is directly related to /1/, where the problem of stabilization of a stochastic system with several perturbations and availability of complete information was reduced to the problem of optimization of a simpler system with a lesser number of perturbations. That idea is further developed here for application in cases of incomplete information.

The problem of optimal control construction for a system with additive perturbations and incomplete information is readily solved by using the theorem on separation /2/. In the case of multiplicative perturbations the optimal filter becomes nonlinear and, generally, infinite dimensional /3/. This creates mathematical difficulties in the investigation of its possibilities in the problem of constructing stabilizing controls. Such controls designed on the principle of direct feedback based on observed variables were used in /4,5/ for systems with multiplicative perturbations and incomplete information. However such controls often do not stabilize a system even in the determinate case.

The necessary condition of stability of a system with perturbations is its stability in the absence of perturbations. Hence it is reasonable to seek controls for stabilizing stochastic systems among controls that stabilize determinate systems. This is taken below as the starting point.

1. Statement of the problem. Consider the stochastic system S_k

$$x^{\cdot} = Ax + Bu + \sum_{r=1}^{k} \gamma_r(x, u) v_r^{\cdot}, \quad \gamma_r(x, u) = \sqrt{x^* Q_r x + u^* P_r u} \tag{1.1}$$

with incomplete information, when observation is defined by the stochastic system

$$y^{\cdot} = Cx + \sum_{r=1}^{k} \gamma_r(x, u) w_r^{\cdot} \tag{1.2}$$

where x is an n-dimensional vector, u is the m-dimensional control, y is the q-dimensional vector of observed variables; A, B and C are matrices of dimensions $n \times n$, $n \times m$, $q \times n$, $v_r(t)$ and $w_r(t)$ are Wiener vector processes independent in the aggregate, of dimensions n and q with parameters

$$M\{v_r(t) - v_r(s)\} = 0, \; M\{[v_r(t) - v_r(s)][v_r(t) - v_r(s)]^*\} = V_r \cdot |t - s|$$

$$M\{w_r(t) - w_r(s)\} = 0, \; M\{[w_r(t) - w_r(s)][w_r(t) - w_r(s)]^*\} = W_r \cdot |t - s|$$

and Q_r, P_r, V_r, W_r are nonnegative determinate matrices $(Q_r \geqslant 0, \; P_r \geqslant 0, \; V_r \geqslant 0, \; W_r \geqslant 0)$ of

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To stabilize system (1.1), (1.2) we shall use a control consisting of a filter (of the known Kalman—Bucy structure), viz.

$$z^{\cdot} = Az + Bu + L(y^{\cdot} - Cz) \tag{1.3}$$

and of feedback

$$u = -Kz \tag{1.4}$$

The dynamic properties of control (1.3), (1.4) depend on the selection of matrices K and L of dimensions $m \times n$ and $n \times q$.

If the pair (A, B) can be stabilized and the pair (A, C) can be detected (condition A), then by an appropriate selection of K and L it is always possible to have control (1.3), (1.4) such that the determinate system

$$x^{\cdot} = Ax + Bu, \; y^{\cdot} = Cx \tag{1.5}$$

is stabilized (see, e.g., /6/).

2. Stability. Consider besides system

$$x^{\cdot} = Ax + \sum_{r=1}^{p} \alpha_r(x) v_r^{\cdot}, \quad \alpha_r(x) = \sqrt{x^* Q_r x} \tag{2.1}$$

the system

$$x^{\cdot} = Ax + \sum_{r=1}^{p} \alpha_r(x) v_r^{\cdot} + \alpha(x) v^{\cdot}, \quad \alpha(x) = \sqrt{x^* Q x} \tag{2.2}$$

with the additional perturbation $\alpha(x) v^{\cdot}$, where $v(t)$ is an independent in the aggregate with $v_r(t)$ n-dimensional Wiener process with parameters

$$M\{v(t) - v(s)\} = 0, \; M\{[v(t) - v(s)][v(t) - v(s)]^*\} = V \cdot |t - s|$$

Theorem 2.1. If system (2.1) is exponentially stable in the mean square, then for system (2.2) to be exponentially stable in the mean square it is necessary and sufficient that the following inequality be satisfied:

$$I = M \int_0^{\infty} \alpha^2(x)\, dt < 1$$

where $x(t)$ is the solution of system (2.1) for the random initial vector $x(0)$ for which $M\{x(0)x^*(0)\} = V$.

This condition was used in /1/ for investigating the problem of stabilization in the case of complete information, although it was not explicitly formulated there. Its proof based on the method of Liapunov's functions using spectral properties of positive operators is, in essence, fully contained in the proof of Theorem 2.1 in /1/. Note that stability criteria close to the above appeared in /7,8/. In the case of incomplete information problem of stabilization can be conveniently solved using another criterion of stability.

Besides (2.1) and (2.2) we consider the system

$$x^{\cdot} = Ax + \sum_{r=1}^{p} \alpha_r(x) v_r^{\cdot} + v^{\cdot} \tag{2.3}$$

obtained from (2.2) by substituting for the additional multiplicative perturbation $\alpha(x) v^{\cdot}$ the respective additive perturbation $v^{\cdot}$.

Let $M(t) = M\{x(t) x^*(t)\}$ be the matrix of second moments of solution $x(t)$ of system (2.1). and $D(t) = M\{x(t) x^*(t)\}$ is the matrix of second moments of solution $x(t)$ of system (2.3).

Using Ito's formula it is possible to show that these matrices satisfy the determinate linear differential equations

$$M^{\cdot} = L(M), \; D^{\cdot} = L(D) + V, \quad L(M) = AM + MA^* + \sum_{r=1}^{p} \operatorname{tr}(Q_r M) V_r$$

where $\operatorname{tr} H$ is the trace of matrix H.

Let $M(0) = V$, $D(0) = 0$. Then

$$D(t) = \int_0^t \exp[(t-s)L](V)\,ds = \int_0^t M(\tau)\,d\tau \tag{2.4}$$

If system (2.1) is stable (exponentially in the mean square), then system (2.3) has a unique steady distribution state x_s (see /9,10/), and $M\{x_s x_s^*\} = \lim D(t)$ as $t \to \infty$. Using equality (2.4) and the readily verified equality $M\{x^*Qx\} = \mathrm{tr}\,(QM\{xx^*\})$, we now obtain

$$M\{\alpha^2(x_s)\} = M\{x_s^* Q x_s\} = \lim_{t\to\infty} \mathrm{tr}\,(QD(t)) = \int_0^\infty \mathrm{tr}\,(QM(\tau))\,d\tau = I$$

A similar transition for systems without multiplicative perturbations appeared in /11/. As the result, we obtain from Theorem 2.1 the following criterion.

Theorem 2.2. If system (2.1) is stable, then for system (2.2) to be stable it is necessary and sufficient that the inequality $M\{\alpha^2(x_s)\} < 1$ is satisfied.

Cases in which the perturbation parameters are not exactly known are often encountered in practice. For instance, only rough estimates of perturbation intensity may be obtained. In such cases rough sufficient conditions which enable us to evaluate the system stability by simpler methods are of considerable interest. We propose to use in such cases the theorem of merging.

Theorem 2.3. Besides system (2.1) we consider the system

$$x^\cdot = Ax + \alpha(x)\,v^\cdot, \quad \alpha(x) = \sqrt{x^* Qx} \tag{2.5}$$

Let the relations

$$Q_r \leqslant Q \; (r = 1, 2, \ldots, p), \quad v(t) = \sum_{r=1}^{p} v_r(t)$$

be satisfied. Then from the stability of system (2.5) follows the stability of system (2.1). This theorem can be proved by using, for instance, the method of Liapunov's functions /9/.

Remark. The device of perturbation majoration can be used also in the case of system

$$x^\cdot = Ax + \sum_{r=1}^{p} \sigma_r x \zeta_r^\cdot \tag{2.6}$$

where σ_r are constant $(n \times n)$ matrices, and $\zeta_r(t)$ are independent in the aggregate standard scalar Wiener processes.

As the majorating system for (2.6) we can take (2.5) in which $v(t) = \sqrt{p}\, v_0(t)$ where $v_0(t)$ is an n-dimensional process whose components are independent standard Wiener processes and matrix Q is such that

$$\sigma_r^* \sigma_r \leqslant Q \quad (r = 1, 2, \ldots, p)$$

3. Stabilization of a process with a single perturbation.

Consider system (1.1), (1.2) in the case of $k = 1$

$$x^\cdot = Ax + Bu + \gamma(x, u)\,v^\cdot \tag{3.1}$$

$$y^\cdot = Cx + \gamma(x, u)\,w^\cdot, \quad \gamma(x, u) = (x^*Qx + u^*Pu)^{1/2}$$

We use control (1.3), (1.4) for stabilizing system (3.1). We then obtain a closed system in variables $x(t)$ and $z(t)$ which is conveniently written in the form

$$\mathbf{X}^\cdot = \mathbf{A}(R)\mathbf{X} + (\mathbf{X}^*\mathbf{Q}(R)\mathbf{X})^{1/2}\,\mathbf{v}^\cdot \tag{3.2}$$

where

$$\mathbf{X} = \left\| \begin{matrix} x \\ z \end{matrix} \right\|, \quad \mathbf{A}(R) = \left\| \begin{matrix} A & -BK \\ LC & A - BK - LC \end{matrix} \right\|, \quad R = [K, L]$$

$$\mathbf{Q}(R) = \left\| \begin{matrix} Q & 0 \\ 0 & K^*PK \end{matrix} \right\|, \quad \mathbf{v} = \left\| \begin{matrix} v(t) \\ Lw(t) \end{matrix} \right\|$$

Consider the set $\mathbf{R} = \{R \mid$ matrix $\mathbf{A}(R)$ is stable.

Under condition A we have $\mathbf{R} \neq \varnothing$.

To stabilize system (3.1) using control (1.3), (1.4) means to select parameters R so as to make system (3.2) stable. According to Theorem 2.2 for system (3.2) with $R \in \mathbf{R}$ to be stable it is necessary and sufficient that the inequality

$$I(R) = M\{\mathbf{X}^*(R)\ \mathbf{Q}(R)\ \mathbf{X}(R)\} < 1$$

where $X(R)$ is the steady distribution state of system

$$\mathbf{X}^{\cdot} = \mathbf{A}(R)\mathbf{X} + \mathbf{v}^{\cdot}$$

is satisfied.

It is obvious that then the necessary and sufficient condition of system (3.1) stabilization by control (1.3), (1.4) is the fulfillment of the inequality

$$\inf_{R \in \mathbf{R}} I(R) < 1$$

The inverse substitution implies that the optimized functional

$$I(R) = M\{x_R^* Q x_R + z_R^* K^* PK z_R\} = M\{\gamma^2(x_R, u_R)\}$$

is determined for the steady distribution state x_R of system (3.3) with observation of (3.4)

$$x^{\cdot} = Ax + Bu + v^{\cdot} \tag{3.3}$$
$$y^{\cdot} = Cx + w^{\cdot} \tag{3.4}$$

and control u_R shaped by control (1.3), (1.4) with parameter R. As the result, we have the following theorem.

Theorem 3.1. If the determinate system (1.5) can be stabilized by control (1.3), (1.4) $(\mathbf{R} \neq \varnothing)$, then for system (3.1) to be stabilizable by control (1.3), (1.4) it is necessary and sufficient that the inequality

$$I = \inf_{R \in \mathbf{R}} M\{\gamma^2(x_R, u_R)\} < 1$$

where x_R is the steady distribution state of system (3.3) with observation of (3.4) and control u_R. Any control (1.3), (1.4) with parameters $R \in \mathbf{R}$ for which $M\{\gamma^2(x_R, u_R)\} < 1$ will then stabilize system (3.1).

Because system (3.3), (3.4) contains only additive perturbations, it is possible to use the theorem of separation /2/, which enables us to readily solve the arising optimization problem.

We restrict the analysis to the case of nondegenerate perturbations, and assume that matrices Q, P, V, W are positive definite $(Q > 0, P > 0, V > 0, W > 0)$. The optimal control parameters K_0 and L_0 are obtained from the relations

$$K_0 = P^{-1}B^*D, \quad L_0 = SC^*W^{-1} \tag{3.5}$$

where $D > 0$ and $S > 0$ are solutions of equations

$$A^*D + DA - DBP^{-1}B^*D = -Q, \ AS + SA^* - SC^*W^{-1} \times CS = -V \tag{3.6}$$

The existence and uniqueness of solutions of the matrix equations (3.6) is implied by conditions A (see /6/). The optimal value of the functional is

$$I = \mathrm{tr}(DV) + \mathrm{tr}(DBP^{-1}B^*DS) \tag{3.7}$$

As the result, we find that the inequality $I < 1$ is the necessary and sufficient condition of stabilizability of system (3.1); hence control (1.3), (1.4) with parameters (3.5) stabilizes system (3.1).

The quantity $I_1 = \mathrm{tr}(DV)$ is the optimal value of functional $M\{\gamma^2(x, u)\}$ in the problem of optimal control of system (3.3) using a control of the form $u = -Kx$.

The inequality $I_1 < 1$ is the necessary and sufficient condition of stabilizability of system (3.1) with complete information /1/. Hence the quantity $I_2 = \mathrm{tr}(DBP^{-1} \cdot B^*DS)$ may be considered as some addition arising in the case of incomplete information. When the perturbations are nondegenerate, I_2 is strictly positive. As the intensity of perturbations in the channel increases, I_2 increases, which inhibits stabilization $(I_1 + I_2 \geqslant 1)$. A decrease of their intensity results in the decrease of I_2 which in degenerate limit cases may vanish. The latter would indicate that incomplete information does not inhibit stabilization of the system. This occurs in the case of n-th order equations with a single observable unpertrubed coordinate, considered in /12/.

4. Stabilization of a system with several perturbations. Sequential procedure. The determination of stabilizability of a system with several perturbations and construction of the stabilizing control for it is generally achieved by using a sequential procedure similar to that applied in /1/ in the case of complete information. The effect of each sequential perturbation added to the system is analyzed at every step of the procedure. Let us investigate the transition from system S_p to S_{p+1} (for definition of these symbols see (1.1), (1.2)). Let the set $\mathbf{R}_p = \{R \mid \text{control (1.3), (1.4) stabilizes system } S_p\}$ be nonempty. Then, using the stability criterion of Theorem 2.2, it is possible to prove that the necessary and sufficient condition of stabilizability of system S_{p+1} by the control (1.3), (1.4) is the fulfillment of the inequality

$$\inf_{R\in\mathbf{R}_p} M\{\gamma_{p+1}^2(x_R, u_R)\} < 1$$

where x_R is the steady distribution state of system

$$x^{\cdot} = Ax + Bu + \sum_{r=1}^{p} \gamma_r(x, u)v_r^{\cdot} + \dot{v}_{p+1}$$

$$y^{\cdot} = Cx + \sum_{r=1}^{p} \gamma_r(x, u)w_r^{\cdot} + \dot{w}_{p+1}$$

with control u_R formed by control (1.3),(1.4). Besides, the stabilization of system S_{p+1} shall be by control (1.3), (1.4) with parameters $R \in \mathbf{R}_p$ for which $M\{\gamma_{p+1}^2(x, u)\} < 1$.

According to that criterion each step of the procedure reduces to solving the problem of minimization of the functional for a system in which the additional multiplicative perturbation is replaced by an additive one. Application of this procedure consists of solving a sequence of optimization problems. At the first step we have the problem of minimizing the quadratic criterion for a system that contains only additive perturbations, which enables us to apply the theorem of separation (see Sect.3) for the determination of optimal control parameters. Subsequent steps represent qualitatively more complex problems, since they involve the optimization of systems containing in addition to additive also multiplicative perturbations whose presence makes it impossible to obtain the analog of the separation theorem.

5. Majorization of several perturbations by a single perturbation. The sufficient criterion of stabilizability. The determination of necessary and sufficient conditions of stabilizability of a system with several multiplicative perturbations necessitates the application of the sequential procedure. If, however, the problem is limited to the determination of sufficient conditions only, the analysis of stabilizability is considerably simplified by applying the concept of majorization.

Theorem 5.1. Let the perturbations of systems (1.1), (1.2) and (3.1) be linked by the relations

$$v(t) = \sum_{r=1}^{k} v_r(t), \quad w(t) = \sum_{r=1}^{k} w_r(t), \quad \gamma_r(x, u) \leqslant \gamma(x, u) \ (r = 1,2,\ldots, k) \tag{5.1}$$

Then from the stabilizability of system (3.1) follows the stabilizability of system (1.1), (1.2), and control (1.3), (1.4) with parameters K and L which stabilizes system (3.1) will also stabilize system (1.1), (1.2). The proof of Theorem 5.1 directly follows from Theorem 2.3, if one takes into account that perturbations of the closed system (3.1), (1.3), (1.4) majorate by virtue of (5.1) the perturbations of the closed system (1.1), (1.2), (1.3), (1.4).

The inequality $I < 1$ (see (3.7)) which by Theorem 3.1 is the necessary and sufficient condition of stabilizability of system (3.1) is, thus, the sufficient condition of stabilizability of system (1.1), (1.2). When $I < 1$, then control (1.3), (1.4) with parameters (3.5) that stabilizes system (3.1) will also stabilize system (1.1), (1.2).

Thus, while the determination of necessary and sufficient conditions of stabilizability of system (1.1), (1.2) by the sequential procedure requires the solution of k optimization problems, the determination of sufficient conditons necessitates the solution of only one such problem. Note that from the point of view of computation the problem reduces to solving matrix equations (3.6), well known in connection with the problem of analytic construction of controls. This also simplifies the determination of the respective stabilizing control.

6. Example. Consider the system

$$x_1^{\cdot} = x_2 + (\sigma_1 x_1 + \sigma_2 x_2)\xi_1^{\cdot},\ x_2^{\cdot} = u + \varphi x_2 \xi_2^{\cdot} + \vartheta u \eta_1^{\cdot} \tag{6.1}$$

whose behavior can be judged by values of y,

$$y^{\cdot} = x_1 + \psi x_1 \xi_3^{\cdot} + \beta u \eta_2^{\cdot} \tag{6.2}$$

We construct for system (6.1), (6.2) a majorating system of the form

$$\begin{aligned} &x_1^{\cdot} = x_2 + \gamma(x_1, x_2, u)\zeta_1^{\cdot},\ x_2^{\cdot} = u + \gamma(x_1, x_2, u)\zeta_2^{\cdot} \\ &y^{\cdot} = x_1 + \gamma(x_1, x_2, u)\zeta_3^{\cdot} \\ &\gamma(x_1, x_2, u) = (x^* Q x + P u^2)^{1/2},\ x^* = (x_1, x_2) \end{aligned} \tag{6.3}$$

where $\zeta_1(t)$, $\zeta_2(t)$, $\zeta_3(t)$ are independent in the aggregate standard Wiener processes.

For system (6.3) to be majorating for system (6.1), (6.2), it is sufficient to set, for example,

$$Q = \begin{Vmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{Vmatrix} = \begin{Vmatrix} \sigma_1^2 + \psi^2 & \sigma_1\sigma_2 \\ \sigma_1\sigma_2 & \sigma_2^2 + \varphi^2 \end{Vmatrix},\quad P = \max\{\vartheta^2, \beta^2\} \tag{6.4}$$

To stabilize systems (6.1), (6.2) and (6.3) we shall use control (1.3), (1.4) which in this case is of the form

$$\begin{aligned} &z_1^{\cdot} = z_2 + l_1(y^{\cdot} - z_1),\ z_2^{\cdot} = u + l_2(y^{\cdot} - z_1) \\ &u = -k_1 z_1 - k_2 z_2 \end{aligned} \tag{6.5}$$

Let us determine the necessary and sufficient conditions of stabilizability of system (6.3). For this (see Sect.3) we substitute for the multiplicative perturbations the corresponding additive perturbations and for the obtained system

$$x_1^{\cdot} = x_2 + \zeta_1^{\cdot},\ x_2^{\cdot} = u + \zeta_2^{\cdot},\ y^{\cdot} = x_1 + \zeta_3^{\cdot} \tag{6.6}$$

solve the optimization problem using the criterion

$$I(u) = M\{\gamma^2(x_1, x_2, u)\} \tag{6.7}$$

For problem (6.6), (6.7) the control (6.5) is optimal with parameters

$$k_1 = d_{12}/P,\ k_2 = d_{22}/P,\ l_1 = s_{11},\ l_2 = s_{12}$$

(see (3.5), (3.6)), where d_{12}, d_{22} and s_{11}, s_{12} are elements of matrices $D > 0$ and $S > 0$ which are solutions of equations

$$A^* D + DA - \frac{1}{P} D b b^* D = -Q,\quad AS + SA^* - Scc^* S = -E$$

$$A = \begin{Vmatrix} 0 & 1 \\ 0 & 0 \end{Vmatrix},\quad b = \begin{Vmatrix} 0 \\ 1 \end{Vmatrix},\quad c = \begin{Vmatrix} 1 \\ 0 \end{Vmatrix},\quad E = \begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix}$$

which in this case are readily solved

$$\begin{aligned} &d_{12} = (Pq_{11})^{1/2},\quad d_{22} = [P(q_{22} + 2d_{12})]^{1/2},\quad d_{11} = \frac{1}{P} d_{12} d_{22} - q_{12} \\ &s_{11} = \sqrt{3},\quad s_{12} = 1,\quad s_{22} = \sqrt{3} \end{aligned} \tag{6.8}$$

The optimal value of criterion $I(u)$ is

$$I = d_{11} + d_{22} + \frac{1}{P}(s_{11} d_{12}^2 + 2 s_{12} d_{12} d_{22} + s_{22} d_{22}^2)$$

With allowance for (6.4) and (6.8) we can obtain for I an explicit expression in terms of parameters $\sigma_1, \sigma_2, \varphi, \psi, \nu, \beta$ of the input system. Since the inequality $I < 1$ is the necessary and sufficient condition of system (6.3) stabilizability, it is the sufficient condition of stabilizability of system (6.1), (6.2). If $I < 1$, control (6.5) with parameters

$$k_1 = \left(\frac{q_{11}}{P}\right)^{1/2},\quad k_2 = \left[\frac{q_{22} + 2(Pq_{11})^{1/2}}{P}\right]^{1/2},\quad l_1 = \sqrt{3},\quad l_2 = 1$$

where q_{11}, q_{22} and P are obtained from (6.4), stabilizes system (6.3) and, consequently also system (6.1), (6.2).

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